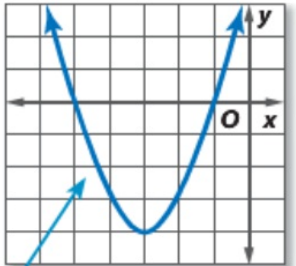
4-8 Graph Quadratic Inequalities

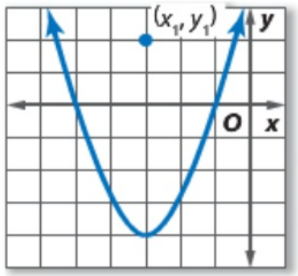
1 Graph Quadratic Inequalities You can graph **quadratic inequalities** in two variables by using the same techniques used to graph linear inequalities in two variables.

Step 1 Graph the related function.



Should the parabola be solid or dashed?

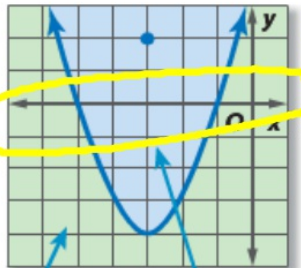
Step 2 Test a point not on the parabola.



$$y_1 \overset{?}{\geq} a(x_1)^2 + b(x_1) + c$$

Is (x_1, y_1) a solution?

Step 3 Shade accordingly.



(x_1, y_1) is a solution.

(x_1, y_1) is not a solution.

If I let y = 0....

Then I can focus on the x-axis!

Example 1 Graph a Quadratic Inequality

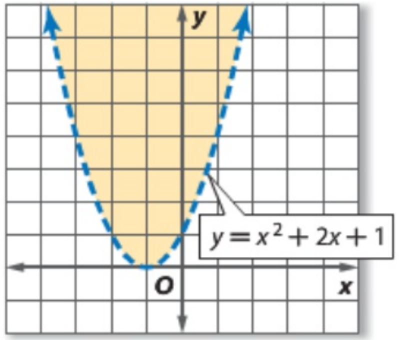
Graph $y > x^2 + 2x + 1$.

Step 1 Graph the related function, $y = x^2 + 2x + 1$. The parabola should be dashed.

Step 2 Test a point not on the graph of the parabola.

$$y > x^2 + 2x + 1$$
$$-1 \overset{?}{>} 0^2 + 2(0) + 1$$
$$-1 \not> 1$$

So, $(0, -1)$ is *not* a solution of the inequality.



Step 3 Shade the region that does not contain the point $(0, -1)$.

Example 1 Graph each inequality. **1–3. See margin.**

1. $y \le x^2 - 8x + 2$
2. $y > x^2 + 6x - 2$
3. $y \ge -x^2 + 4x + 1$

$x = \frac{-(-8)}{2(1)} = \frac{8}{2} = 4$

$x = 4$

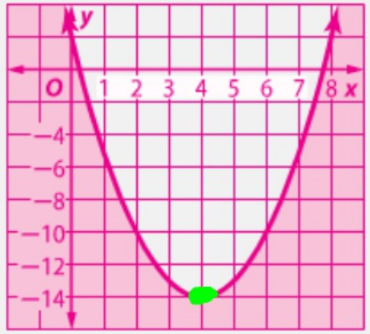
$y = 16 - 32 + 2 = -14$

$(4, -14)$

vertex

Additional Answers

1.



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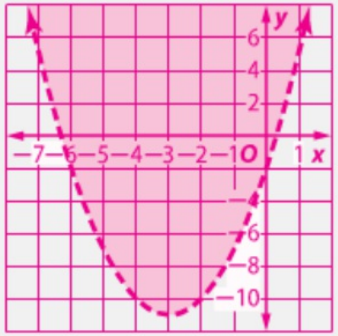

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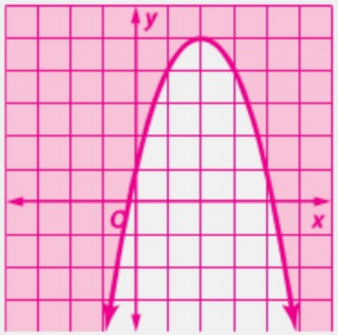


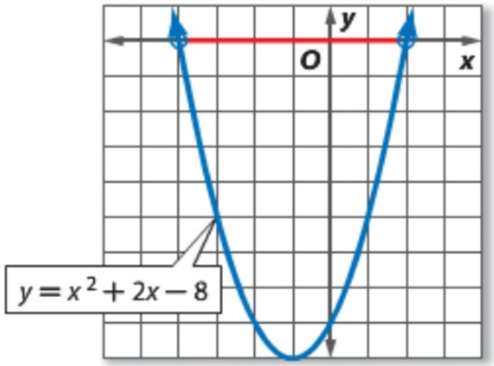
Example 2 Solve $ax^2 + bx + c < 0$ by Graphing

Solve $x^2 + 2x - 8 < 0$ by graphing.

The solution consists of x-values for which the graph of the related function lies *below* the x-axis. Begin by finding the roots of the related function.

$x^2 + 2x - 8 = 0$	Related equation
$(x - 2)(x + 4) = 0$	Factor.
$x - 2 = 0$ or $x + 4 = 0$	Zero Product Property
$x = 2$ $\quad$ $x = -4$	Solve each equation.

Sketch the graph of a parabola that has x-intercepts at -4 and 2. The graph should open up because $a > 0$.



The graph lies below the x-axis between $x = -4$ and $x = 2$. Thus, the solution set is $\{x \mid -4 < x < 2\}$ or $(-4, 2)$.

CHECK Test one value of x less than -4, one between -4 and 2, and one greater than 2 in the original inequality.

Test $x = -6$.	**Test $x = 0$.**	**Test $x = 5$.**
$x^2 + 2x - 8 < 0$	$x^2 + 2x - 8 < 0$	$x^2 + 2x - 8 < 0$
$(-6)^2 + 2(-6) - 8 \overset{?}{<} 0$	$0^2 + 2(0) - 8 \overset{?}{<} 0$	$5^2 + 2(5) - 8 \overset{?}{<} 0$
$16 < 0$ ✗	$-8 < 0$ ✓	$27 < 0$ ✗

Example 3 Solve $ax^2 + bx + c \geq 0$ by Graphing

Solve $2x^2 + 4x - 5 \geq 0$ by graphing.

The solution consists of x-values for which the graph of the related function lies *on and above* the x-axis. Begin by finding the roots of the related function.

$2x^2 + 4x - 5 = 0$ — Related equation

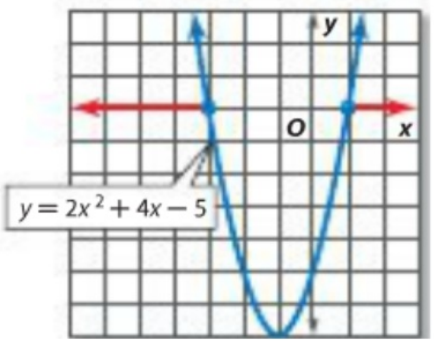
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Use the Quadratic Formula

$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}$ — Replace a with 4, b with 2, and c with −5.

$x = \frac{-4 + \sqrt{56}}{4}$ or $x = \frac{-4 - \sqrt{56}}{4}$ — Simplify and write as two equations.

≈ 0.87 $\quad\approx -2.87$ — Simplify.

Sketch the graph of a parabola with x-intercepts at −2.87 and 0.87. The graph opens up since $a > 0$. The graph lies on and above the x-axis at about $x \leq -2.87$ and $x \geq 0.87$. Therefore, the solution is approximately $\{x \mid x \leq -2.87 \text{ or } x \geq 0.87\}$ or $(-\infty, -2.87] \cup [0.87, \infty)$.



Examples 2–3 CCSS **SENSE-MAKING** **Solve each inequality by graphing.**

4. $0 < x^2 - 5x + 4$ $\{x \mid x < 1 \text{ or } x > 4\}$

5. $x^2 + 8x + 15 < 0$ $\{x \mid -5 < x < -3\}$

6. $-2x^2 - 2x + 12 \geq 0$ $\{x \mid -3 \leq x \leq 2\}$

7. $0 \geq 2x^2 - 4x + 1$ $\{x \mid 0.29 \leq x \leq 1.71\}$

(4) $x^2 - 5x + 4 > 0$

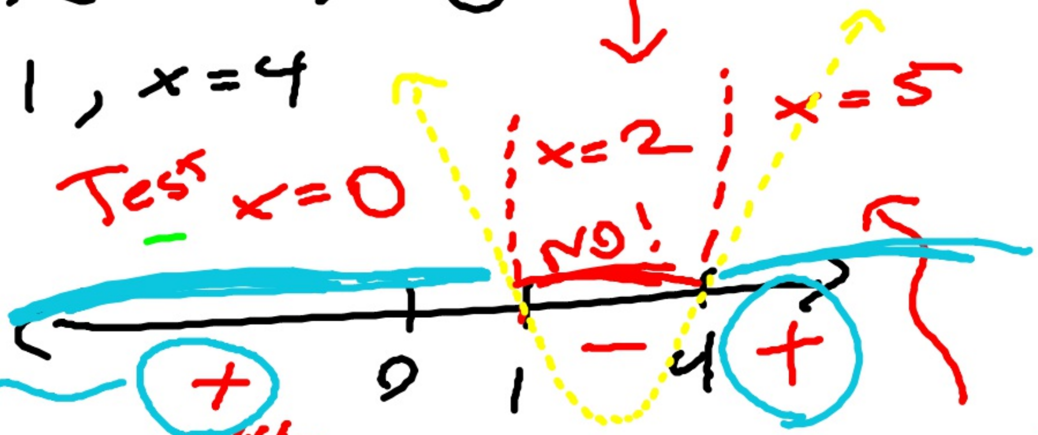
$(x-1)(x-4) = 0$

$x = 1,\ x = 4$

$2^2 - 5(2) + 4$

$4 - 10 + 4$

Test $x = 0$



true

$0 - 0 + 4 = 4$

$5^2 - 5(5) + 4$

$25 - 25 + 4$

Examples 2–3 CCSS **SENSE-MAKING** **Solve each inequality by graphing.**

4. $0 < x^2 - 5x + 4$ $\{x \mid x < 1 \text{ or } x > 4\}$

5. $x^2 + 8x + 15 < 0$ $\{x \mid -5 < x < -3\}$

6. $-2x^2 - 2x + 12 \geq 0$ $\{x \mid -3 \leq x \leq 2\}$

7. $0 \geq 2x^2 - 4x + 1$ $\{x \mid 0.29 \leq x \leq 1.71\}$

7) $2x^2 - 4x + 1 \leq 0$

$a = 2$
$b = -4$
$c = 1$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4}$$

$$x = 1 \pm \frac{\sqrt{2}}{4}$$

$0 + 0 + 1 \leq 0$

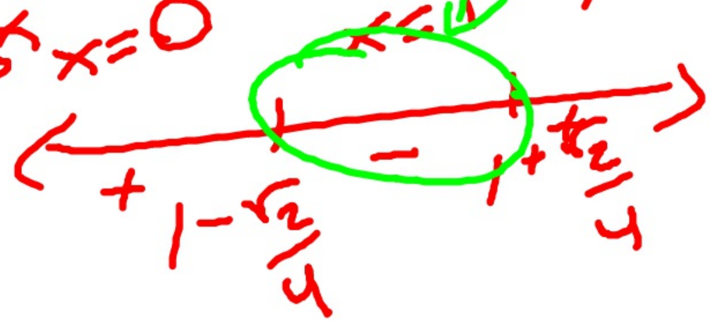
1

test $x = 0$

$2 - 4 + 1$

$x = 5$

$50 - 20 + 1$



Real-World Example 4 Solve a Quadratic Inequality

WATER BALLOONS Refer to the beginning of the lesson. At what time will a water balloon be within 3 meters of the ground after it has been launched?

The function $h(t) = -4.9t^2 + 32t + 1.2$ describes the height of the water balloon. Therefore, you want to find the values of t for which $h(t) \leq 3$.

$h(t) \leq 3$	Original inequality
$-4.9t^2 + 32t + 1.2 \leq 3$	$h(t) = -4.9t^2 + 32t + 1.2$
$-4.9t^2 + 32t - 1.8 \leq 0$	Subtract 3 from each side.

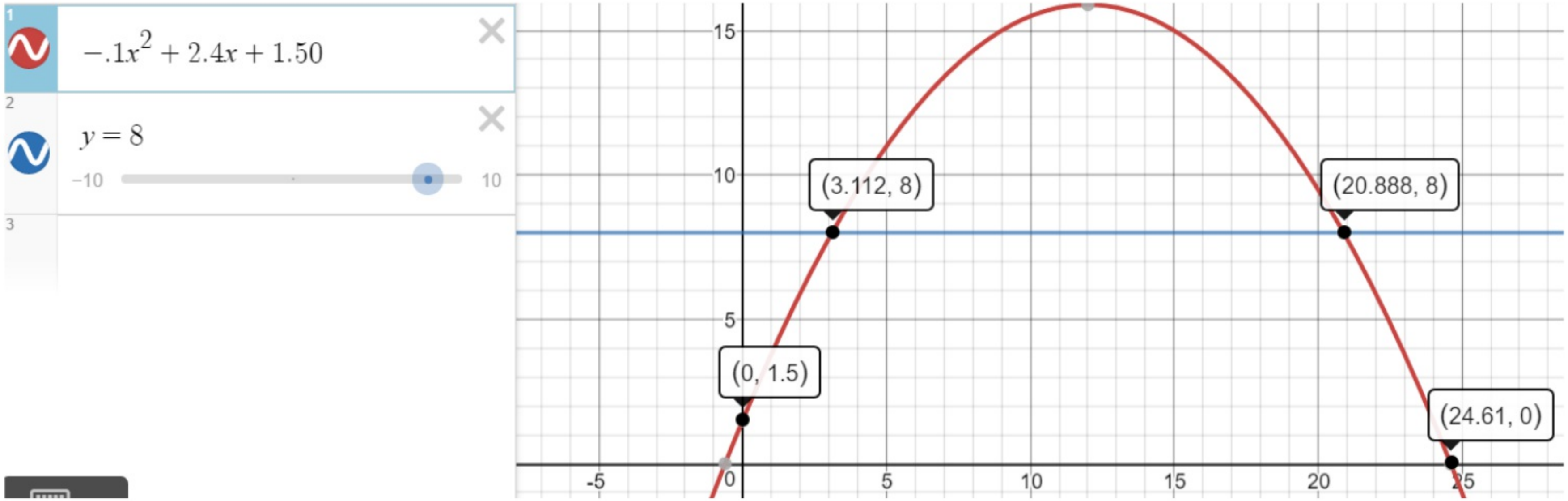
Graph the related function $y = -4.9x^2 + 32x - 1.8$ using a graphing calculator. The zeros of the function are about 0.06 and 6.47, and the graph lies below the x-axis when $x < 0.06$ and $x > 6.47$.

So, the water balloon is within 3 meters of the ground during the first 0.06 second after being launched and again after about 6.47 seconds until it hits the ground.

[−1, 9] scl: 1 by [−5, 55] scl: 5

Example 4

8. **SOCCER** A midfielder kicks a ball toward the goal during a match. The height of the ball in feet above the ground $h(t)$ at time t can be represented by $h(t) = -0.1t^2 + 2.4t + 1.5$. If the height of the goal is 8 feet, at what time during the kick will the ball be able to enter the goal? **$\{t \mid 0 < t < 3.11\}$ or $\{t \mid 20.89 < t \leq 24.61\}$**



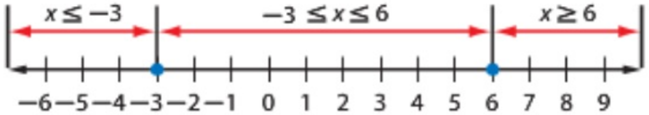
Example 5 Solve a Quadratic Inequality Algebraically

Solve $x^2 - 3x \leq 18$ algebraically.

Step 1 Solve the related quadratic equation $x^2 - 3x = 18$.

$x^2 - 3x = 18$	Related quadratic equation
$x^2 - 3x - 18 = 0$	Subtract 18 from each side.
$(x + 3)(x - 6) = 0$	Factor.
$x + 3 = 0$ or $x - 6 = 0$	Zero Product Property
$x = -3$ $\quad$ $x = 6$	Solve each equation.

Step 2 Plot -3 and 6 on a number line. Use dots since these values are solutions of the original inequality. Notice that the number line is divided into three intervals.



Step 3 Test a value from each interval to see if it satisfies the original inequality.

$\boldsymbol{x \leq -3}$	$\boldsymbol{-3 \leq x \leq 6}$	$\boldsymbol{x \geq 6}$
Test $x = -5$.	Test $x = 0$.	Test $x = 8$.
$x^2 - 3x \leq 18$	$x^2 - 3x \leq 18$	$x^2 - 3x \leq 18$
$(-5)^2 - 3(-5) \stackrel{?}{\leq} 18$	$(0)^2 - 3(0) \stackrel{?}{\leq} 18$	$(8)^2 - 3(8) \stackrel{?}{\leq} 18$
$40 \nleq 18$	$0 \leq 18$	$40 \nleq 18$

The solution set is $\{x \mid -3 \leq x \leq 6\}$ or $[-3, 6]$.

Example 5 Solve each inequality algebraically.

9. $x^2 + 6x - 16 < 0$ $\{x \mid -8 < x < 2\}$
10. $x^2 - 14x > -49$ $\{x \mid x < 7 \text{ or } x > 7\}$
11. $-x^2 + 12x \geq 28$ $\{x \mid 3.17 \leq x \leq 8.83\}$
12. $x^2 - 4x \leq 21$ $\{x \mid -3 \leq x \leq 7\}$

Practice and Problem Solving

Extra Practice is on page R4.

Example 1 Graph each inequality. 13–18. See margin.

13. $y \geq x^2 + 5x + 6$
14. $x^2 - 2x - 8 < y$
15. $y \leq -x^2 - 7x + 8$
16. $-x^2 + 12x - 36 > y$
17. $y > 2x^2 - 2x - 3$
18. $y \geq -4x^2 + 12x - 7$

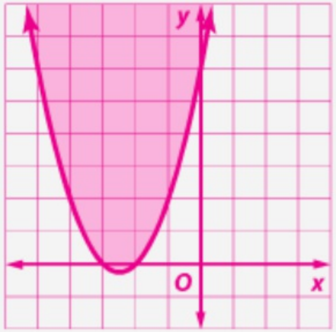
Examples 2–3 Solve each inequality by graphing.

19. $x^2 - 9x + 9 < 0$
20. $x^2 - 2x - 24 \leq 0$
21. $x^2 + 8x + 16 \geq 0$
22. $x^2 + 6x + 3 > 0$
23. $0 > -x^2 + 7x + 12$
24. $-x^2 + 2x - 15 < 0$
25. $4x^2 + 12x + 10 \leq 0$ $\varnothing$
26. $-3x^2 - 3x + 9 > 0$
27. $0 > -2x^2 + 4x + 4$
28. $3x^2 + 12x + 36 \leq 0$ $\varnothing$
29. $0 \leq -4x^2 + 8x + 5$ $\{x \mid -0.5 \leq x \leq 2.5\}$
30. $-2x^2 + 3x + 3 \leq 0$ $\{x \mid x \leq -0.69 \text{ or } x \geq 2.19\}$

19. $\{x \mid 1.1 < x < 7.9\}$
20. $\{x \mid -4 \leq x \leq 6\}$
21. {all real numbers}
22. $\{x \mid x < -5.45 \text{ or } x > -0.55\}$
23. $\{x \mid x < -1.42 \text{ or } x > 8.42\}$
24. {all real numbers}
26. $\{x \mid -2.30 < x < 1.30\}$
27. $\{x \mid x < -0.73 \text{ or } x > 2.73\}$

Additional Answers

13.



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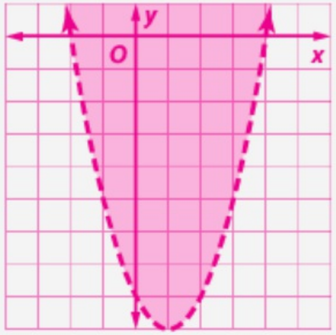

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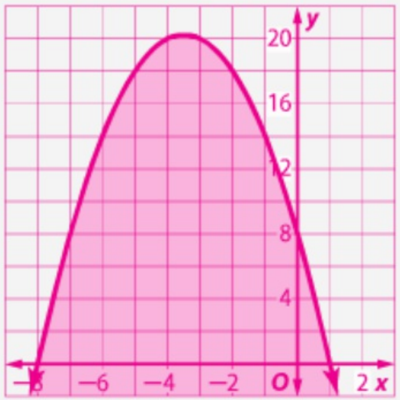

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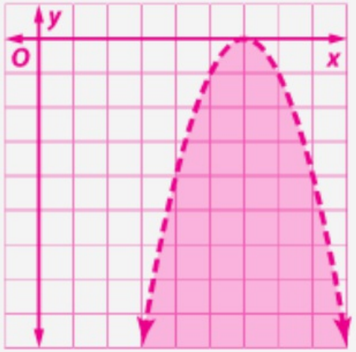

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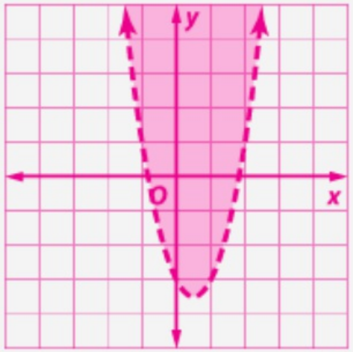

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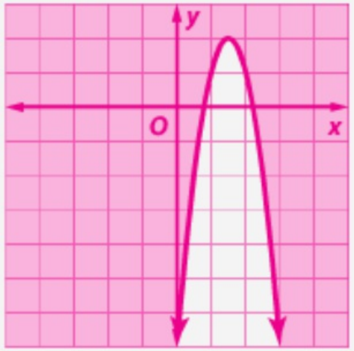


24. {all real numbers}
26. $\{x \mid -2.30 < x < 1.30\}$
27. $\{x \mid x < -0.73$ or $x > 2.73\}$

19. $x^2 - 9x + 9 < 0$ **20.** $x^2 - 2x - 24 \leq 0$ **21.** $x^2 + 8x + 16 \geq 0$

22. $x^2 + 6x + 3 > 0$ **23.** $0 > -x^2 + 7x + 12$ **24.** $-x^2 + 2x - 15 < 0$

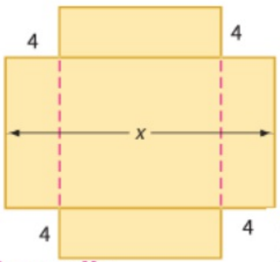
25. $4x^2 + 12x + 10 \leq 0$ $\varnothing$ **26.** $-3x^2 - 3x + 9 > 0$ **27.** $0 > -2x^2 + 4x + 4$

28. $3x^2 + 12x + 36 \leq 0$ $\varnothing$ **29.** $0 \leq -4x^2 + 8x + 5$ $\{x \mid -0.5 \leq x \leq 2.5\}$ **30.** $-2x^2 + 3x + 3 \leq 0$ $\{x \mid x \leq -0.69$ or $x \geq 2.19\}$

Example 4

31 **ARCHITECTURE** An arched entry of a room is shaped like a parabola that can be represented by the equation $f(x) = -x^2 + 6x + 1$. How far from the sides of the arch is its height at least 7 feet? **about 1.26 ft to 4.73 ft**

32. **MANUFACTURING** A box is formed by cutting 4-inch squares from each corner of a square piece of cardboard and then folding the sides. If $V(x) = 4x^2 - 64x + 256$ represents the volume of the box, what should the dimensions of the original piece of cardboard be if the volume of the box cannot exceed 750 cubic inches? **greater than 8 in. but no more than 21.69 in.**



Example 5

Solve each inequality algebraically. **33–44. See Chapter 4 Answer Appendix.**

33. $x^2 - 9x < -20$ **34.** $x^2 + 7x \geq -10$ **35.** $2 > x^2 - x$

36. $-3 \leq -x^2 - 4x$ **37.** $-x^2 + 2x \leq -10$ **38.** $-6 > x^2 + 4x$

39. $2x^2 + 4 \geq 9$ **40.** $3x^2 + x \geq -3$ **41.** $-4x^2 + 2x < 3$

42. $-11 \geq -2x^2 - 5x$ **43.** $-12 < -5x^2 - 10x$ **44.** $-3x^2 - 10x > -1$

Lesson 4-8

33. $\{x \mid 4 < x < 5\}$

34. $\{x \mid x \leq -5$ or $x \geq -2\}$

35. $\{x \mid -1 < x < 2\}$

36. $\{x \mid -4.65 \leq x \leq 0.65\}$

37. $\{x \mid x \leq -2.32$ or $x \geq 4.32\}$

38. $\varnothing$

39. $\{x \mid x \leq -1.58$ or $x \geq 1.58\}$

40. {all real numbers}

41. {all real numbers}

42. $\{x \mid x \leq -3.91$ or $x \geq 1.41\}$

43. $\{x \mid -2.84 < x < 0.84\}$

44. $\{x \mid -3.43 < x < 0.10\}$